Satellite Model for Yaw-Axis Determination and Control Using PID Compensator

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ABSTRACT

The attitude determination and control of satellites are some of the most important subsystems of a satellite since the accuracy of its mission depends on this subsystem. The control task of this subsystem demands an optimal controller capable of accurately rotating the satellite body into desired attitude. In this paper, a PID controller was designed as the compensator to step the yaw-axis angle of the satellite body through the accurate angular position of a DC motor with optimum response as permissible. A mathematical model for the whole satellite yaw angle control system has been derived. Again, the powerful computational tool of MATLAB was used to perfect the controller design and verify the result obtained. Thus, the design of control system for the yaw angle with the fastest settling time, 1.09seconds, 4.55% overshoot and final value of 1 were achieved.

Keywords: Satellite, Attitude Control System, Stability Analysis, PID, Compensator, MATLAB.

1.0 INTRODUCTION

The orientation in space with respect to different coordinate systems is referred to as the satellite attitude [1]. Real-time or post-facto knowledge, and maintenance of a desired, specified attitude within a given tolerance in a satellite system is known as the attitude determination and control. Attitude determination and control of satellites (ADCS) are some of the most important subsystems of a satellite. This is because the accuracy of its mission depends on the subsystem. It is the satellites visual sense and feeling in space especially in small satellites [1].

Furthermore, previous researches have been made on the various methods and reasons of attitude control of the satellites. Some of the reasons for attitude control designs are as follow [2]:

- In communication satellites, antennas are required to be focused to a certain point on earth with high accuracy.
- In earth observing satellites, cameras are required to be focused to a certain point on earth to give an acceptable video coverage of the region.
- In orbital maneuver, desired attitude must be maneuvered.
- For maximum use of solar energy, solar cells must be positioned to the sun.

As mentioned earlier, different researches have been reported on the design of attitude control system such as the Orsted satellite attitude control system (ACS) project or the 65kg microsatellite [3]. The Princeton satellite system project paper together with CTA space systems incorporates the

development of satellite attitude control system architecture, called the SPACE CRAFT CONTROL SYSTEM [3, 4, and 5]. However, this research paper reports the mobilization of a very powerful design and computational tool, MATLAB in optimal design for satellite ACS.

This paper presents a complete mathematical model for Low Earth Orbit (LEO) satellite control system. A DC motor has been technically selected as the actuating elements to rotate the satellite body to the desired yaw angle [6, 3]. The optimal control objective of fastest response time 1.09seconds and 4.55% overshoot has been achieved utilizing the correcting signal from the saturated amplifier and PID controller. A MATLAB Program was written for the PID controller design and the stability analysis conducted using the Nyquist stability criterion.

2.0 DESIGN OBJECTIVES.

Attitude Control means placing the satellite in a specific predetermined direction. This consists of attitude stabilization and control maneuver. The satellite is assumed to be a rigid body operating in frictionless space with the disturbances. As a resulting of these disturbances, in time, the satellite will drift from the desired attitude angle, θ (t). Thus, a suitable control system that returns and maintains the satellite on its desired attitude angle, θ (t) = 0 must be designed. The control system response should be as fast as possible with minimal overshoot and zero- steady state error. Below in figure 1 is a schematic representation of the overall control systems.

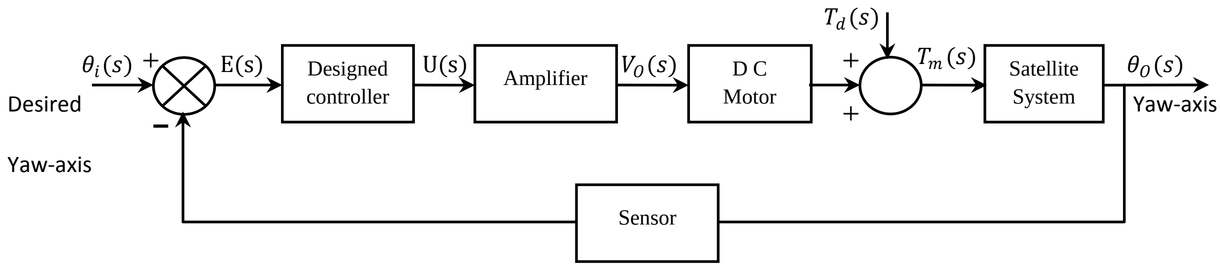

Figure 1: Yaw-axis attitude control system.

3.0 DESIGN SPECIFICATIONS

The control design specifications are given below;

- The settling time is to be ≤ 2 seconds,
- The overshoot $\leq 5\%$ and
- Zero steady-state error.

4.0 SATELLITE ATTITUDE CONTROL SYSTEM MATHEMATICAL MODELS.

Mathematical models of physical systems are key elements in the design and analysis of control systems. To understand and control the complex satellite system, a quantitative mathematical model of the system must be derived from basic relationship between system variables.

Hence, for the complete mathematical model of satellite ADCS, a model of the individual control elements consisting of the Amplifier, DC motor and the satellite system are developed in the subsequent sections. Figure 2 below is an algorithm for the proposed models.

4.1 AMPLIFIER MATHEMATICAL MODELS

Amplifiers used in many applications have gain, k_a. The output voltage, $V_o(s)$ is

$$V_o(s) = k_a V_i(s) \quad (1a)$$

Thus, the open-loop transfer function yields,

$$\frac{V_o(s)}{V_i(s)} = k_a \quad (1b)$$

4.2 DC MOTOR MATHEMATICAL MODEL

The DC motor is the power actuator device that delivers output torque, $T_m(s)$ from the motor. The input is the amplifier voltage output, $V_o(s)$ which supplies current, I_a to the resistance, R_a and inductance, L_a of the armature windings. The input voltage may be modeled in terms of the field or armature terminals. Here, we make use of the armature – controlled DC motor which uses I_a as the control current [7]. From figure 3, when a constant field current is established in the field coil, the motor torque, $T_m(s)$ is;

$$T_m(s) = K_m I_a(s) \quad (2)$$

The armature current is related to the input voltage applied to the armature by

$$V_a(s) = (R_a + L_a S) I_a(s) + V_b(s) \quad (3)$$

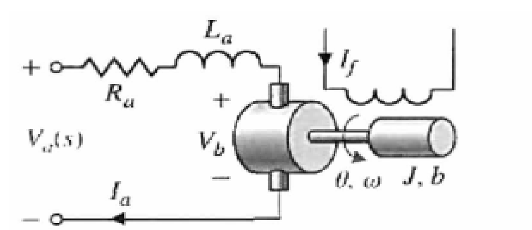

Figure 3: DC motor armature-controlled rotational actuator.

Where $V_b(s)$ is the back electromotive force volt proportional to the motor speed.

Therefore, $V_b(s) = K_b \omega(s)$ (4)

Where $\omega(s) = S\,\theta(s)$ which is the transform of the angular speed and the armature current is

$$I_a(s) = \frac{V_a(S) - K_b \omega(s)}{R_a + L_a S} \quad (5)$$

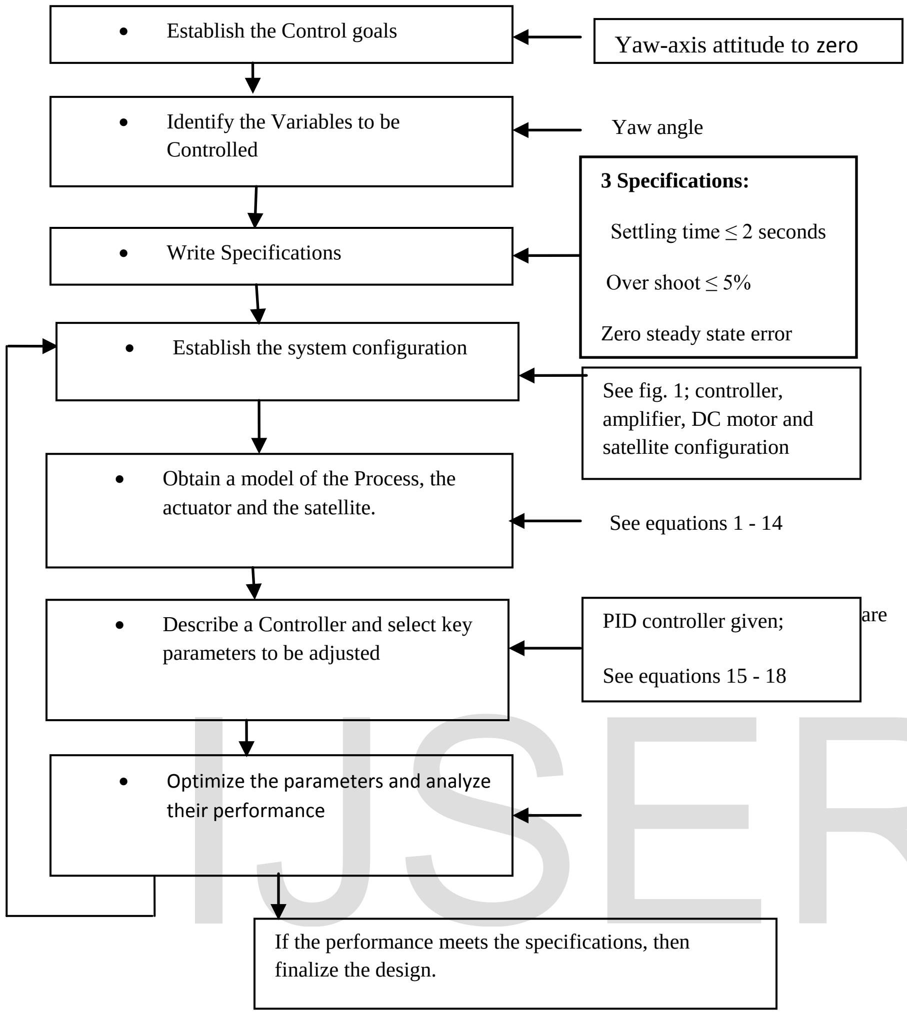

Fig. 2: Design Algorithm for the ADCS.

Again, the motor torque $T_m(s)$ is equal to the torque delivered to the load hence,

$$T_m(s) = T_L(s) + T_d(s) \quad (6)$$

Where $T_d(s)$ is the disturbance torque. The load torque, T_L for rotating inertia as shown in figure 3 is written as,

$$T_L(s) = s(Js + b)\theta(s) \quad (7)$$

Re-arranging equations (2) – (5), we obtain

$$T_L(s) = T_m(s) - T_d(s) = s(Js + b)\theta(s) \quad (8)$$

For most DC motors, $k_m = k_b = K$, Combining equations (2), (5) and (8) and letting $T_d(s) = 0$, we obtain the D.C motor transfer function as [7],

$$\frac{\theta(s)}{V_o(s)} = \frac{K}{(Js + b)(R_a + L_a S) + K^2} \quad (9)$$

4.3 SATELLITE SYSTEM MATHEMATICAL MODEL

The moment of inertia of the entire system is J^1 which encompasses both the satellite body moment of inertia about the axis of rotation at the centre of mass (J_f) and that of motor (J_a). There is a vicious friction, B, as part of the load elements. The angular displacement $\theta_o(s)$ of the satellite body around the yaw-axis is the output of the satellite system and DC motor

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torque $T_m(s)$ is the input. Thus the transfer function of the satellite attitude system is $\theta_o(s)/T_m(s)$. The differential equation for the load elements is,

$$J^1\left(\frac{d^2\theta_o}{dt^2}\right) + B\left(\frac{d\theta_o}{dt}\right) = T_m \qquad (10)$$

Taking the Laplace transform of both sides of the equation (10) and assuming zero initial conditions we get;

$$J^1(s^2\theta_0(s)) + B(s\theta_0(s)) = T_m(s) \qquad (11)$$

Rearranging Equations (11) yields;

$$\frac{\theta_0(s)}{T_m(s)} = \frac{1}{s(J^1 s + B)} \qquad (12)$$

Hence, the combination of equations (1), (9) and (12) yields the overall transfer function of the complete satellite attitude model with U(s) as the control input signal;

$$\frac{\theta_0(s)}{U(s)} = \frac{K}{(Js+b)(R_a+L_a S)+K^2} * \frac{1}{s(J^1 s + B)} \qquad (13)$$

Equation (13) represents the satellite system transfer function for type one system. In the subsequent sections, for the design of the optimal controller, the parameters of the satellite body inertia, the D.C motor gains armature, inertia, electrical and electronic components, and amplifier gain are of utmost importance and affects the whole design process. Thus, the following typical parameters for D.C motors, amplifier gain and LEO satellite according to [7, 8 and 9], were assumed in this design.

Table 1: ADCS for satellite parameters [7,8,9].

Amplifier constant, k_a	10
DC Motor constant, K	0.01Nm/Amp
Armature resistance, R_a	1ohms
Armature Inductance, L_a	0.5H
DC motor moment of inertia, J	$0.01kgm^2$
Motor damping ratio, b	0.1Nms
Satellite moment of inertia, J^1	$2.5kgm^2$ *
Satellite damping ratio, B	1.17N-ms *

Hence substituting the parameters into equation (13) yields the open-loop transfer function as;

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{0.1}{s(0.025s^2 + 0.262s + 0.117)} \qquad (14)$$

The plot of equation (14) to a step response is as shown below in figure 4 and the following step data were obtained with the aid of the powerful tool, MATLAB. Rise time: 4.705, settling time: 8.4719, settling min: 0.7729, settling max: 0.8541, overshoot: 0%, Undershoot: 0% peak: 0.854, peak time: 15.6655. From in figure 4 response, it is imperative that a controller be designed in order to meet with the system design specifications.

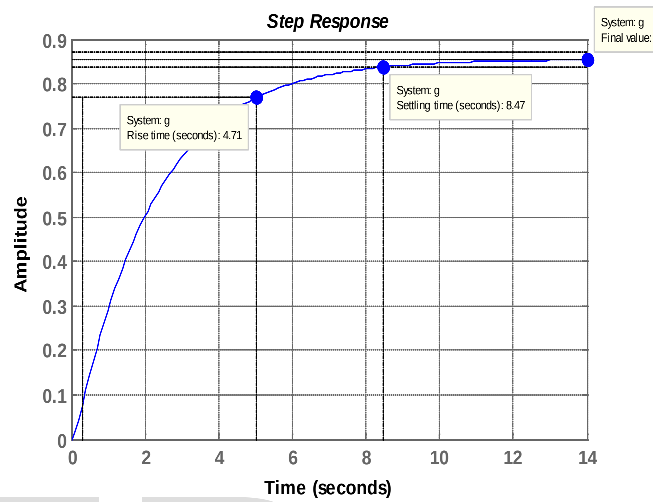

Figure 4: Open-loop step plot for the Yaw-axis of ADCS.

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5.0 PID CONTROLLER DESIGN

The three term proportional-plus-integral-plus-derivative, PID controller is one form of controller most widely used in the optimal process control and since the satellite system contains an integrator, equation (14) makes it the most desirable choice of controller [10,11]. The controller has the transfer function;

$$G_C(s) = K_P + K_I/_S + K_D S \qquad (15)$$

It is called a PID because it contains proportional, K_P, integral, K_I, and derivative, K_D terms. From equation (15),

$$G_C(s) = \frac{K_D S^2 + K_P S' + KI'}{S} = K_D(S^2 + aS + a) \qquad (16)$$

Therefore, $G_C(s) = \frac{K_D(S+a)^2}{S}$ (17)

Consequently, from (17), it will be observed that a PID controller introduces a transfer function with one pole at the origin and two zeros that can be located anywhere in the S-plane [11]. The controller is commonly referred to as the second method of the Ziegler-Nichols tuning rule. Thus, the control design problem now is to determine the values of **K** and **a** such that the optimal design specifications are achieved.

A MATHLAB program was written to set the search region as; $2 \le K \le 40$ and $0.05 \le a \le 0.5$.

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The step size for, ***K***, to be 1 and that for, ***a***, to be 0.05, so as to find the first set of variable ***K*** and ***a*** that will satisfy the satellite attitude control specifications. The closed-loop transfer function of the controller and the satellite system $G_C G_S(s)$ is given by;

$$\frac{\theta_{o(s)}}{\theta_{i(s)}} = \frac{(0.1k)s^2 + (0.2ka)s + (0.1ka^2)}{0.025s^4 + 0.262s^3 + (0.117 + (0.1k))s^2 + (0.2ka)s + 0.1ka^2} \quad (18)$$

A possible MATLAB program that produces the first optimal set of variables, ***K*** and ***a*** that satisfies the given specifications are shown in the appendix. The optimal values obtained by this program are; $K = 15, a = 0.15$, overshoot, $m = 4.55\%$ $and\ time, ts = 1.09$ seconds. The resulting unit-step response curve, root locus plot and bode plots of the yaw angle are shown below in figures 5, 6 and 7 respectively.

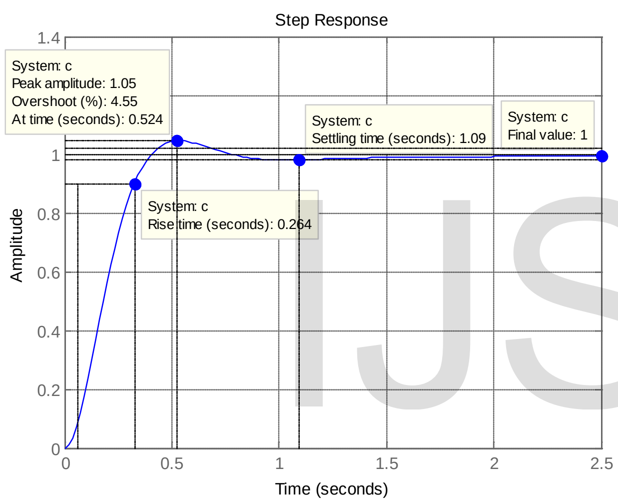

Figure 5: The yaw-axis ADCS unit step response.

From figure 5, improvements in the unit step response parameters are obtained, settling time, t_s reduced from 8.47199 to 1.09seconds, rise time, t_r reduced from 4.705 to 0.264 with a final value of 1. An increase in overshoot from 0% to 4.55% was observed in the above yaw angle step response. The increase in overshoot is as a result of the proportional (K_P) and integral (K_I) actions of the PID controller. Hence, the control design objectives are met.

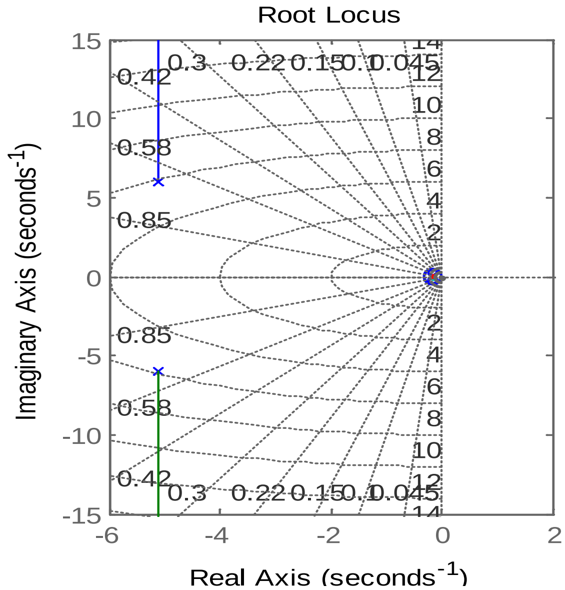

Figure 6: Root locus of the yaw-axis ACS system.

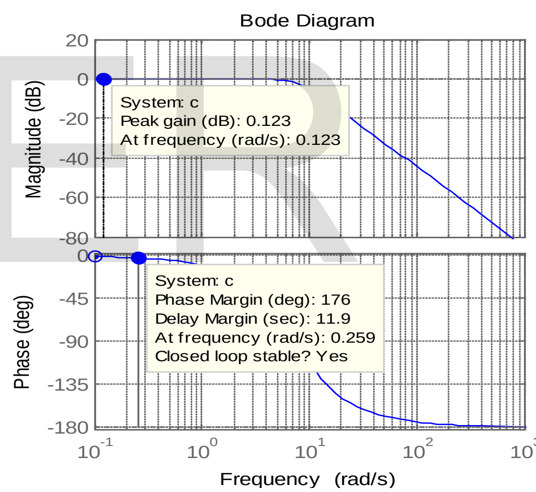

Figure 7: Bode plot of the yaw-axis ACS system.

From the bode plots of the overall system response, it is observed that the system is closed loop stable with phase margin (deg.): 176, frequency (rads/sec): 0.259.

Furthermore, the designed optimal controller satellite attitude system may confidently be verified further by the stability analysis using the Nyquist diagram. Thus, the open loop gain is given below;

$$G_C G_S(s) = \frac{0.1[k(s+a)^2]}{s^2[0.025s^2 + 0.262s + 0.117]} \quad (19)$$

Substituting the optimal values (K, a) of the PID controller in the above equation (19), and rewriting it into a polynomial form, we get;

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$$G_C G_S(s) = \frac{(1.5)s^2 + (0.45)s + (0.034)}{[0.025s^4 + 0.262s^3 + 0.117s^2]} \quad (20)$$

Again, the Nyquist plot is shown in figure 8 below as obtained from running the MATLAB code written in appendix;

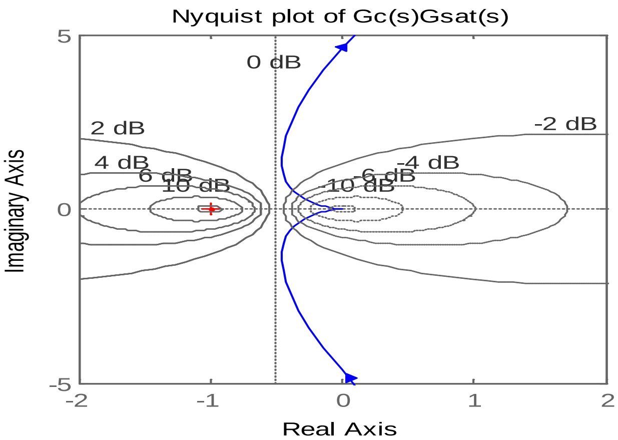

Figure 8: Nyquist plot of $G_C G_S(s)$

Since the (-1) point is not encircled, the system is stable, which is confirmed by the obtained step response of the yaw angle satellite attitude control system, as the unit step response exhibits the maximum overshoot, m, of 4.55%, the settling time, t_s, is 1.09seconds, and final value of 1.

6.0 CONCLUSION

This paper presents the design of a PID as an optimal controller for the satellite attitude control system. Thus, the satellite body can be oriented around the desired Yaw-axis attitude of 90 degrees as quickly as 1.09 seconds settling time without excessive overshoot of not more than 4.55%. A mathematical model for the satellite attitude control system is derived with that of the PID controller.

Again, the powerful MATLAB tool was utilized for the design and stability analysis stage. From the bode plots of the overall system response, it is observed that the system is closed loop stable with phase margin (deg.): 176, frequency (rads/sec): 0.259. Also, the MATHLAB program that computes an optimal PID controller parameters (**k, a**) to suite the control requirements of any satellite attitude control system was developed and implemented.

APPENDIX

% PID CONTROLLER PROGRAMME FOR YAW-AXIS ACS SYSTEM %

```
t=0:0.01:2.5;
for k = 40:-1:2;%start outer loop to vary the k values
for a = 0.5:-0.05:0.05;%start inner loop to vary the k values
num = [0 0 0.1*k 0.2*k*a 0.1*k*a^2];
den = [0.025 0.262 0.117+(0.1*k) 0.2*k*a 0.1*k*a^2];
y = step(num,den,t);
m = max(y);
s = 251; while y(s)>0.98&y(s)< 1.02;
s = s-1;end;
ts = (s-1)*0.01;
if m<1.05 & m > 1.00 & ts < 2.0
break;% breaks the inner loop
end
end
if m<1.05 & m > 1.00 & ts < 2.0
break;% breaks the outer loop
end
end
plot(t,y); grid; title( 'The Satellite Attitude Control System Unit Step Response')
xlabel('Time sec'); ylabel(' The satellite angular yaw-axis position')
solution = [k;a;m;ts]
```

% MATLAB ROOTS LOCUS COMMAND %

```
num= [0 0 1.5 0.45 0.034];
den = [0.025 0.262 1.612 0.45 0.034];
c=tf(num,den);
rlocusplot(c);grid
```

%MATLAB BODE PLOT COMMAND%

num= [0 0 1.5 0.45 0.034];

den = [0.025 0.262 1.612 0.45 0.034];

c=tf(num,den);

bodeplot(c);grid

% Nyquist plot of the SAC system open-loop transfer function %

% $G_C G_S(s) = \frac{(1.5)s^2 + (0.45)s + (0.034)}{[0.025s^4 + 0.262s^3 + 0.117s^2]}$ **%**

num = [0 0 1.5 0.45 0.034];

den = [0.025 0.262 0.117 0 0];

nyquist(num,den)

v = [-2 2 -5 5];axis(v);grid

title('Nyquist plot of Gc(s)Gsat(s)')

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